Locality and beyond: from algebraic quantum field theory to effective quantum gravity

Kasia Rejzner

University of York

Local Quantum Physics and beyond

in memoriam Rudolf Haag,

26.09.2016

Outline of the talk

1. Algebraic approach to QFT
   - AQFT
   - LCQFT
   - pAQFT
2. Quantum gravity
   - Effective quantum gravity
   - Observables

The father of Local Quantum Physics

- Rudolf Haag (1922 – 2016).

The father of Local Quantum Physics

- Rudolf Haag (1922 – 2016).
- The author of a beautiful book *Local Quantum Physics*.

The father of Local Quantum Physics

- Rudolf Haag (1922 – 2016).
- The author of a beautiful book *Local Quantum Physics*.
- One of the fathers of LQP.

The father of Local Quantum Physics

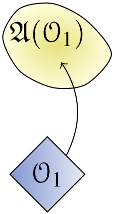
- Rudolf Haag (1922 – 2016).
- The author of a beautiful book *Local Quantum Physics*.
- One of the fathers of LQP.
- We will all miss him. . .

Local Quantum Physics

- Algebraic Quantum Field Theory (Local Quantum Physics) is a mathematically rigorous framework, which allows to investigate conceptual problems in QFT.

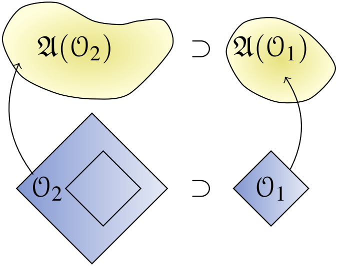
Local Quantum Physics

- Algebraic Quantum Field Theory (Local Quantum Physics) is a mathematically rigorous framework, which allows to investigate conceptual problems in QFT.
- It started as the axiomatic framework of Haag-Kastler [Haag & Kastler 64]: a model is defined by associating to each region $\mathcal{O}$ of Minkowski spacetime the algebra $\mathfrak{A}(\mathcal{O})$ of observables that can be measured in $\mathcal{O}$.

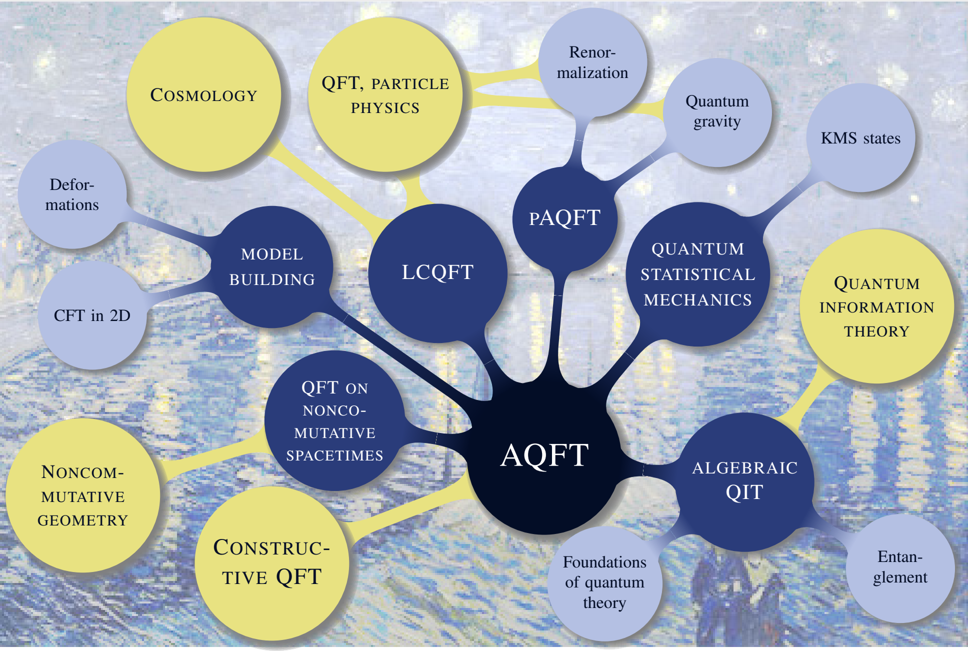


Local Quantum Physics

- Algebraic Quantum Field Theory (Local Quantum Physics) is a mathematically rigorous framework, which allows to investigate conceptual problems in QFT.
- It started as the axiomatic framework of Haag-Kastler [Haag & Kastler 64]: a model is defined by associating to each region $\mathcal{O}$ of Minkowski spacetime the algebra $\mathfrak{A}(\mathcal{O})$ of observables that can be measured in $\mathcal{O}$.
- The physical notion of subsystems is realized by the condition of isotony, i.e.: $\mathcal{O}_2 \supset \mathcal{O}_1 \Rightarrow \mathfrak{A}(\mathcal{O}_2) \supset \mathfrak{A}(\mathcal{O}_1)$. We obtain a net of algebras.



Different aspects of AQFT and relations to physics



Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.

Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.
- The corresponding generalization of AQFT is called locally covariant quantum field theory [Hollands-Wald 01, Brunetti-Fredenhagen-Verch 01, Fewster-Verch 12,. . .].

Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.
- The corresponding generalization of AQFT is called locally covariant quantum field theory [Hollands-Wald 01, Brunetti-Fredenhagen-Verch 01, Fewster-Verch 12,. . .].

Main advantages

Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.
- The corresponding generalization of AQFT is called locally covariant quantum field theory [Hollands-Wald 01, Brunetti-Fredenhagen-Verch 01, Fewster-Verch 12,...].

Main advantages

- Local algebras of observables $\mathfrak{A}(\mathcal{O})$ are defined abstractly, the Hilbert space representation comes later (this deals with the non-uniqueness of the vacuum).

Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.
- The corresponding generalization of AQFT is called locally covariant quantum field theory [Hollands-Wald 01, Brunetti-Fredenhagen-Verch 01, Fewster-Verch 12,...].

Main advantages

- Local algebras of observables $\mathfrak{A}(\mathcal{O})$ are defined abstractly, the Hilbert space representation comes later (this deals with the non-uniqueness of the vacuum).
- Algebras $\mathfrak{A}(\mathcal{O})$ are constructed using only the local data.

Algebraic QFT on curved spacetimes

- Conceptual problems of QFT on curved spacetimes can be easily solved in the algebraic approach, because of the powerful principle of locality.
- The corresponding generalization of AQFT is called locally covariant quantum field theory [Hollands-Wald 01, Brunetti-Fredenhagen-Verch 01, Fewster-Verch 12,...].

Main advantages

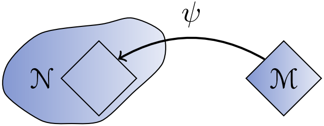
- Local algebras of observables $\mathfrak{A}(\mathcal{O})$ are defined abstractly, the Hilbert space representation comes later (this deals with the non-uniqueness of the vacuum).
- Algebras $\mathfrak{A}(\mathcal{O})$ are constructed using only the local data.
- Local features of the theory (observables) are separated from the global features (states).

Locally covariant quantum field theory (LCQFT)

- In the original AQFT axioms we associate algebras to regions of a fixed spacetimes. Now we go a step further.

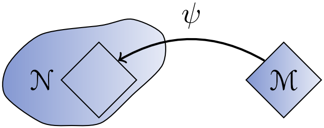
Locally covariant quantum field theory (LCQFT)

- In the original AQFT axioms we associate algebras to regions of a fixed spacetimes. Now we go a step further.
- Replace $\mathcal{O}_1$ and $\mathcal{O}_2$ with arbitrary spacetimes $\mathcal{M} = (M, g)$, $\mathcal{N} = (N, g')$ and require the embedding $\psi : \mathcal{M} \to \mathcal{N}$ to be an isometry.



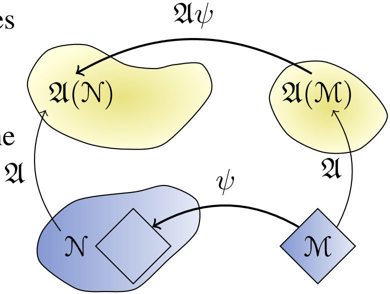
Locally covariant quantum field theory (LCQFT)

- In the original AQFT axioms we associate algebras to regions of a fixed spacetimes. Now we go a step further.
- Replace $\mathcal{O}_1$ and $\mathcal{O}_2$ with arbitrary spacetimes $\mathcal{M} = (M, g)$, $\mathcal{N} = (N, g')$ and require the embedding $\psi : \mathcal{M} \to \mathcal{N}$ to be an isometry.
- Require that ψ preserves orientations and the causal structure (no new causal links are created by the embedding).



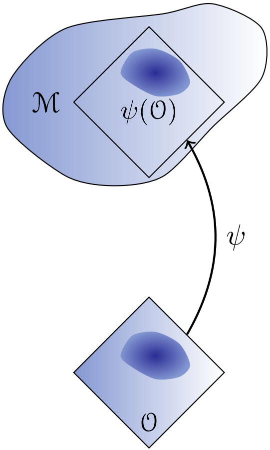
Locally covariant quantum field theory (LCQFT)

- In the original AQFT axioms we associate algebras to regions of a fixed spacetimes. Now we go a step further.
- Replace $\mathcal{O}_1$ and $\mathcal{O}_2$ with arbitrary spacetimes $\mathcal{M} = (M, g)$, $\mathcal{N} = (N, g')$ and require the embedding $\psi : \mathcal{M} \to \mathcal{N}$ to be an isometry.
- Require that ψ preserves orientations and the causal structure (no new causal links are created by the embedding).
- Assign to each spacetime $\mathcal{M}$ an algebra $\mathfrak{A}(\mathcal{M})$ and to each admissible embedding ψ a homomorphism of algebras $\mathfrak{A}\psi$ (notion of subsystems). This has to be done covariantly.



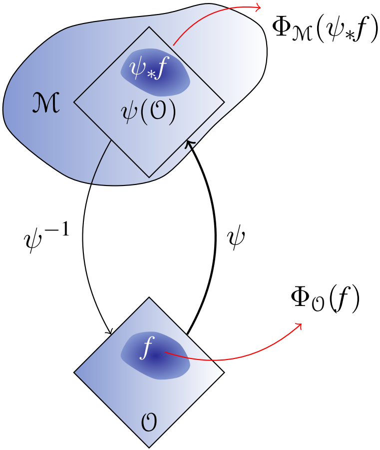
Locally covariant fields

- In the framework of LCQFT, locally covariant fields are used to identify (put labels on) observables localized in different region of spacetime, in the absence of symmetries.



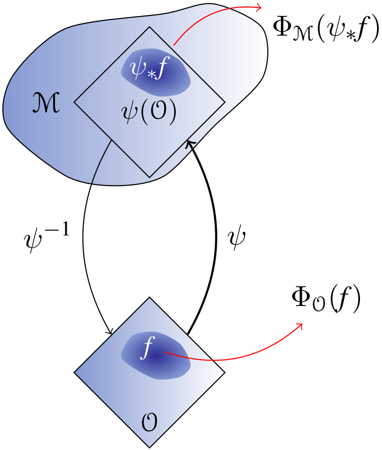
Locally covariant fields

- In the framework of LCQFT, locally covariant fields are used to identify (put labels on) observables localized in different region of spacetime, in the absence of symmetries.
- Let $\mathfrak{D}(\mathcal{O})$ denote the space of test functions supported in $\mathcal{O}$. A locally cov. field is a family of maps $\Phi_{\mathcal{M}} : \mathfrak{D}(\mathcal{M}) \to \mathfrak{A}(\mathcal{M})$, labeled by spacetimes $\mathcal{M}$ such that: $\mathfrak{A}\psi(\Phi_{\mathcal{O}}(f)) = \Phi_{\mathcal{M}}(\psi_* f)$.



Locally covariant fields

- In the framework of LCQFT, locally covariant fields are used to identify (put labels on) observables localized in different region of spacetime, in the absence of symmetries.
- Let $\mathfrak{D}(\mathcal{O})$ denote the space of test functions supported in $\mathcal{O}$. A locally cov. field is a family of maps $\Phi_{\mathcal{M}}: \mathfrak{D}(\mathcal{M}) \to \mathfrak{A}(\mathcal{M})$, labeled by spacetimes $\mathcal{M}$ such that: $\mathfrak{A}\psi(\Phi_{\mathcal{O}}(f)) = \Phi_{\mathcal{M}}(\psi_* f)$.
- This generalizes the notion of Wightman's operator-valued distributions.



Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.
- Main contributions:

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.
- Main contributions:
  - Free theory obtained by the formal deformation quantization of Poisson (Peierls) bracket: $\star$-product ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, . . .]).

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.
- Main contributions:
  - Free theory obtained by the formal deformation quantization of Poisson (Peierls) bracket: $\star$-product ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, ...]).
  - Interaction introduced in the causal approach to renormalization due to Epstein and Glaser ([Epstein-Glaser 73]),

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.
- Main contributions:
  - Free theory obtained by the formal deformation quantization of Poisson (Peierls) bracket: $\star$-product ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, ...]).
  - Interaction introduced in the causal approach to renormalization due to Epstein and Glaser ([Epstein-Glaser 73]),
  - Generalization to gauge theories using homological algebra ([Hollands 08, Fredenhagen-KR 11]).

Perturbative algebraic quantum field theory

- Perturbative algebraic quantum field theory (pAQFT) is a mathematically rigorous framework that allows to build interacting LCQFT models.
- It combines Haag's idea of local quantum physics with methods of perturbation theory.
- Main contributions:
  - Free theory obtained by the formal deformation quantization of Poisson (Peierls) bracket: $\star$-product ([Dütsch-Fredenhagen 00, Brunetti-Fredenhagen 00, Brunetti-Dütsch-Fredenhagen 09, ...]).
  - Interaction introduced in the causal approach to renormalization due to Epstein and Glaser ([Epstein-Glaser 73]),
  - Generalization to gauge theories using homological algebra ([Hollands 08, Fredenhagen-KR 11]).
- For a review see the book: *Perturbative algebraic quantum field theory. An introduction for mathematicians*, KR, Springer 2016.

Physical input

- Spacetime $\mathcal{M} = (M, g)$: a smooth manifold M with a smooth Lorentzian metric g. Assume $\mathcal{M}$ to be oriented, time-oriented and globally hyperbolic (has a Cauchy surface).

Physical input

- Spacetime $\mathcal{M} = (M, g)$: a smooth manifold M with a smooth Lorentzian metric g. Assume $\mathcal{M}$ to be oriented, time-oriented and globally hyperbolic (has a Cauchy surface).
- Configuration space $\mathfrak{E}(\mathcal{M})$: choice of objects we want to study in our theory (scalars, vectors, tensors,...). Typically this is a space of smooth sections of some vector bundle $E \xrightarrow{\pi} M$ over M. For the scalar field: $\mathfrak{E}(\mathcal{M}) = \mathcal{C}^\infty(M, \mathbb{R})$.

Physical input

- Spacetime $\mathcal{M} = (M, g)$: a smooth manifold M with a smooth Lorentzian metric g. Assume $\mathcal{M}$ to be oriented, time-oriented and globally hyperbolic (has a Cauchy surface).
- Configuration space $\mathfrak{E}(\mathcal{M})$: choice of objects we want to study in our theory (scalars, vectors, tensors,...). Typically this is a space of smooth sections of some vector bundle $E \xrightarrow{\pi} M$ over M. For the scalar field: $\mathfrak{E}(\mathcal{M}) = \mathcal{C}^\infty(M, \mathbb{R})$.
- Dynamics: we use a modification of the Lagrangian formalism. Since the manifold M is non-compact, we need to introduce a cutoff function into the Lagrangian. For the free scalar field $L_{\mathcal{M}}(f)(\varphi) = \frac{1}{2} \int (\nabla_\mu \varphi \nabla^\mu \varphi - m^2 \varphi^2)(x) f(x) d\mu_g(x)$.

Physical input

- Spacetime $\mathcal{M} = (M, g)$: a smooth manifold M with a smooth Lorentzian metric g. Assume $\mathcal{M}$ to be oriented, time-oriented and globally hyperbolic (has a Cauchy surface).
- Configuration space $\mathfrak{E}(\mathcal{M})$: choice of objects we want to study in our theory (scalars, vectors, tensors,...). Typically this is a space of smooth sections of some vector bundle $E \xrightarrow{\pi} M$ over M. For the scalar field: $\mathfrak{E}(\mathcal{M}) = \mathcal{C}^\infty(M, \mathbb{R})$.
- Dynamics: we use a modification of the Lagrangian formalism. Since the manifold M is non-compact, we need to introduce a cutoff function into the Lagrangian. For the free scalar field $L_{\mathcal{M}}(f)(\varphi) = \frac{1}{2}\int (\nabla_\mu \varphi \nabla^\mu \varphi - m^2\varphi^2)(x) f(x) d\mu_g(x)$.
- Abstractly, a Lagrangian is a locally covariant classical field (another manifestation of the locality principle).

Observables

- Classical observables are modeled as smooth functionals on the configuration space $\mathfrak{E}(\mathcal{M})$, i.e. elements of $\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$.

Observables

- Classical observables are modeled as smooth functionals on the configuration space $\mathfrak{E}(\mathcal{M})$, i.e. elements of $\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$.
- The support of $F\in\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$ is defined as:

$$\begin{aligned}\operatorname{supp} F = \{x\in M | \forall \text{ neighbourhoods } U \text{ of } x\ \exists \varphi,\psi\in\mathfrak{E}(\mathcal{M}),\\ \operatorname{supp}\psi\subset U \text{ such that } F(\varphi+\psi)\neq F(\varphi)\}\ .\end{aligned}$$

Observables

- Classical observables are modeled as smooth functionals on the configuration space $\mathfrak{E}(\mathcal{M})$, i.e. elements of $\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$.
- The support of $F\in\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$ is defined as:

$$\begin{aligned}\operatorname{supp} F=\{x\in M|\forall \text{ neighbourhoods } U \text{ of } x\ \exists\varphi,\psi\in\mathfrak{E}(\mathcal{M}),\\ \operatorname{supp}\psi\subset U \text{ such that } F(\varphi+\psi)\neq F(\varphi)\}\ .\end{aligned}$$

- F is local if it is of the form: $F(\varphi)=\int_M f(j^k_x(\varphi))\,d\mu(x)$, where f is a function on the jet bundle over M and $j^k_x(\varphi)=(\varphi(x),\partial\varphi(x),\dots)$ (up to order k) is the k-th jet of φ at the point x. Let $\mathfrak{F}_{\mathrm{loc}}(\mathcal{M})$ denote the space of local functionals.

Observables

- Classical observables are modeled as smooth functionals on the configuration space $\mathfrak{E}(\mathcal{M})$, i.e. elements of $\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$.
- The support of $F\in\mathcal{C}^\infty(\mathfrak{E}(\mathcal{M}),\mathbb{R})$ is defined as:

$$\operatorname{supp}F=\{x\in M|\forall \text{ neighbourhoods } U \text{ of } x\ \exists\varphi,\psi\in\mathfrak{E}(\mathcal{M}),\\ \operatorname{supp}\psi\subset U \text{ such that } F(\varphi+\psi)\neq F(\varphi)\}\ .$$

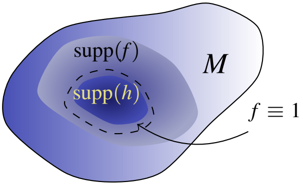
- F is local if it is of the form: $F(\varphi)=\int_M f(j^k_x(\varphi))\,d\mu(x)$, where f is a function on the jet bundle over M and $j^k_x(\varphi)=(\varphi(x),\partial\varphi(x),\dots)$ (up to order k) is the k-th jet of φ at the point x. Let $\mathfrak{F}_{\mathrm{loc}}(\mathcal{M})$ denote the space of local functionals.
- Consider functionals that are multilocal, i.e. they are sums of products of local functionals. Denote them by $\mathfrak{F}(\mathcal{M})$; they play the role of polynomials.

Equations of motion I

- Actions S are equivalence classes of Lagrangians $L_1 \sim L_2$ if $\mathrm{supp}(L_1 - L_2)(f) \subset \mathrm{supp}\, df$.

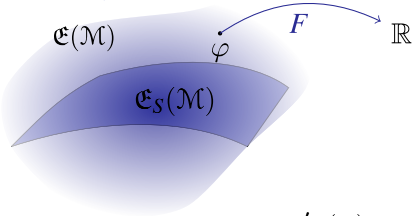
Equations of motion I

- Actions S are equivalence classes of Lagrangians $L_1 \sim L_2$ if $\mathrm{supp}(L_1 - L_2)(f) \subset \mathrm{supp}\, df$.
- The Euler-Lagrange derivative of an ation $S_{\mathcal{M}}$ is a map $S'_{\mathcal{M}} : \mathfrak{E}(\mathcal{M}) \to \mathfrak{D}'(\mathcal{M})$ defined as (thanks to locality): $\left\langle S'_{\mathcal{M}}(\varphi), h \right\rangle = \left\langle L_{\mathcal{M}}(f)^{(1)}(\varphi), h \right\rangle$, where $f \equiv 1$ on supp$h$.



Equations of motion I

- Actions S are equivalence classes of Lagrangians $L_1 \sim L_2$ if $\mathrm{supp}(L_1 - L_2)(f) \subset \mathrm{supp}\, df$.
- The Euler-Lagrange derivative of an ation $S_{\mathcal{M}}$ is a map $S'_{\mathcal{M}} : \mathfrak{E}(\mathcal{M}) \to \mathfrak{D}'(\mathcal{M})$ defined as (thanks to locality): $\left\langle S'_{\mathcal{M}}(\varphi), h \right\rangle = \left\langle L_{\mathcal{M}}(f)^{(1)}(\varphi), h \right\rangle$, where $f \equiv 1$ on supp$h$.



- The equation of motion (EOM) is the equation $S'_{\mathcal{M}}(\varphi) \equiv 0$ for an unknown function $\varphi \in \mathfrak{E}(\mathcal{M})$ and it determines a subspace of $\mathfrak{E}(\mathcal{M})$ denoted by $\mathfrak{E}_S(\mathcal{M})$ (on-shell configurations).

Equations of motion II

- In the algebraic spirit, characterize $\mathfrak{E}_S(\mathcal{M})$ by its space of functions $\mathfrak{F}_S(\mathcal{M})$, given by the quotient $\mathfrak{F}_S(\mathcal{M}) = \mathfrak{F}(\mathcal{M})/\mathfrak{F}_0(\mathcal{M})$, where $\mathfrak{F}_0(\mathcal{M})$ is generated by the elements of the form:
$$\varphi \mapsto \left\langle S'_{\mathcal{M}}(\varphi), X(\varphi) \right\rangle = \partial_X S_{\mathcal{M}}(\varphi),$$
with X a vector field on $\mathfrak{E}(\mathcal{M})$, i.e. $X \in \Gamma(T\mathfrak{E}(\mathcal{M}))$.

Equations of motion II

- In the algebraic spirit, characterize $\mathfrak{E}_S(\mathcal{M})$ by its space of functions $\mathfrak{F}_S(\mathcal{M})$, given by the quotient $\mathfrak{F}_S(\mathcal{M}) = \mathfrak{F}(\mathcal{M})/\mathfrak{F}_0(\mathcal{M})$, where $\mathfrak{F}_0(\mathcal{M})$ is generated by the elements of the form:
$$\varphi \mapsto \left\langle S'_{\mathcal{M}}(\varphi), X(\varphi) \right\rangle = \partial_X S_{\mathcal{M}}(\varphi),$$
with X a vector field on $\mathfrak{E}(\mathcal{M})$, i.e. $X \in \Gamma(T\mathfrak{E}(\mathcal{M}))$.
- Let $\mathfrak{V}(\mathcal{M})$ denote the space of multilocal vector fields on $\mathfrak{E}(\mathcal{M})$ and let $\delta X \doteq \partial_X S_{\mathcal{M}}$.

Equations of motion II

- In the algebraic spirit, characterize $\mathfrak{E}_S(\mathcal{M})$ by its space of functions $\mathfrak{F}_S(\mathcal{M})$, given by the quotient $\mathfrak{F}_S(\mathcal{M}) = \mathfrak{F}(\mathcal{M})/\mathfrak{F}_0(\mathcal{M})$, where $\mathfrak{F}_0(\mathcal{M})$ is generated by the elements of the form:
$$\varphi \mapsto \left\langle S'_{\mathcal{M}}(\varphi), X(\varphi) \right\rangle = \partial_X S_{\mathcal{M}}(\varphi),$$
with X a vector field on $\mathfrak{E}(\mathcal{M})$, i.e. $X \in \Gamma(T\mathfrak{E}(\mathcal{M}))$.
- Let $\mathfrak{V}(\mathcal{M})$ denote the space of multilocal vector fields on $\mathfrak{E}(\mathcal{M})$ and let $\delta X \doteq \partial_X S_{\mathcal{M}}$.
- The kernel of δ consists of vector fields $X \in \mathfrak{V}(\mathcal{M})$, which satisfy $\left\langle S'_{\mathcal{M}}(\varphi), X(\varphi) \right\rangle = 0$ for all $\varphi \in \mathfrak{E}(\mathcal{M})$, i.e. $\partial_X S_{\mathcal{M}} \equiv 0$.

Equations of motion II

- In the algebraic spirit, characterize $\mathfrak{E}_S(\mathcal{M})$ by its space of functions $\mathfrak{F}_S(\mathcal{M})$, given by the quotient $\mathfrak{F}_S(\mathcal{M}) = \mathfrak{F}(\mathcal{M})/\mathfrak{F}_0(\mathcal{M})$, where $\mathfrak{F}_0(\mathcal{M})$ is generated by the elements of the form:
$$\varphi \mapsto \left\langle S'_{\mathcal{M}}(\varphi), X(\varphi)\right\rangle = \partial_X S_{\mathcal{M}}(\varphi),$$
with X a vector field on $\mathfrak{E}(\mathcal{M})$, i.e. $X \in \Gamma(T\mathfrak{E}(\mathcal{M}))$.
- Let $\mathfrak{V}(\mathcal{M})$ denote the space of multilocal vector fields on $\mathfrak{E}(\mathcal{M})$ and let $\delta X \doteq \partial_X S_{\mathcal{M}}$.
- The kernel of δ consists of vector fields $X \in \mathfrak{V}(\mathcal{M})$, which satisfy $\left\langle S'_{\mathcal{M}}(\varphi), X(\varphi)\right\rangle = 0$ for all $\varphi \in \mathfrak{E}(\mathcal{M})$, i.e. $\partial_X S_{\mathcal{M}} \equiv 0$.
- These characterize directions in the configuration space $\mathfrak{E}(\mathcal{M})$ in which the action S is constant, we call them local symmetries.

Symmetries and the kernel of δ

- The space of symmetries includes elements of the form $\delta(\Lambda^2\mathfrak{V}(\mathcal{M}))$, where $\Lambda^2\mathfrak{V}(\mathcal{M})$ is the second exterior power of $\mathfrak{V}(\mathcal{M})$. Such symmetries are called trivial, because they vanish on $\mathfrak{E}_S(\mathcal{M})$. Consider a complex

$$\ldots \to \Lambda^2\mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{F}(\mathcal{M}) \to 0,$$

Symmetries and the kernel of δ

- The space of symmetries includes elements of the form $\delta(\Lambda^2\mathfrak{V}(\mathcal{M}))$, where $\Lambda^2\mathfrak{V}(\mathcal{M})$ is the second exterior power of $\mathfrak{V}(\mathcal{M})$. Such symmetries are called trivial, because they vanish on $\mathfrak{E}_S(\mathcal{M})$. Consider a complex

$$\ldots \to \Lambda^2\mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{F}(\mathcal{M}) \to 0,$$

- Note that $\mathfrak{F}_S(\mathcal{M}) = H_0(\Lambda\mathfrak{V}(\mathcal{M}), \delta)$ and H_1 characterizes non-trivial local symmetries.

Symmetries and the kernel of δ

- The space of symmetries includes elements of the form $\delta(\Lambda^2\mathfrak{V}(\mathcal{M}))$, where $\Lambda^2\mathfrak{V}(\mathcal{M})$ is the second exterior power of $\mathfrak{V}(\mathcal{M})$. Such symmetries are called trivial, because they vanish on $\mathfrak{E}_S(\mathcal{M})$. Consider a complex

$$\ldots \to \Lambda^2\mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{F}(\mathcal{M}) \to 0,$$

- Note that $\mathfrak{F}_S(\mathcal{M}) = H_0(\Lambda\mathfrak{V}(\mathcal{M}), \delta)$ and H_1 characterizes non-trivial local symmetries.
- Working with $\Lambda\mathfrak{V}(\mathcal{M})$ instead of $\mathfrak{F}_S(\mathcal{M})$ allows us to quantize the theory off-shell.

Symmetries and the kernel of δ

- The space of symmetries includes elements of the form $\delta(\Lambda^2\mathfrak{V}(\mathcal{M}))$, where $\Lambda^2\mathfrak{V}(\mathcal{M})$ is the second exterior power of $\mathfrak{V}(\mathcal{M})$. Such symmetries are called trivial, because they vanish on $\mathfrak{E}_S(\mathcal{M})$. Consider a complex

$$\ldots \to \Lambda^2\mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{V}(\mathcal{M}) \xrightarrow{\delta} \mathfrak{F}(\mathcal{M}) \to 0,$$

- Note that $\mathfrak{F}_S(\mathcal{M}) = H_0(\Lambda\mathfrak{V}(\mathcal{M}), \delta)$ and H_1 characterizes non-trivial local symmetries.
- Working with $\Lambda\mathfrak{V}(\mathcal{M})$ instead of $\mathfrak{F}_S(\mathcal{M})$ allows us to quantize the theory off-shell.
- In the presence of non-trivial symmetries, one has to further extend the configuration space and replace δ with s, the classical BV operator.

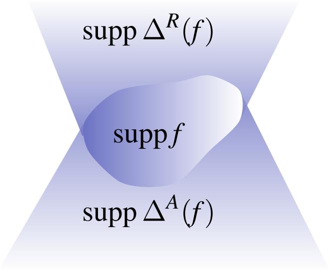
Free scalar field

- For the free scalar field the equation of motion is of the form $S'_{\mathcal{M}}(\varphi) = P\varphi = 0$, where $P = -(\Box + m^2)$ is the Klein-Gordon operator.

Free scalar field

- For the free scalar field the equation of motion is of the form $S'_{\mathcal{M}}(\varphi) = P\varphi = 0$, where $P = -(\Box + m^2)$ is the Klein-Gordon operator.
- If $\mathcal{M}$ is globally hyperbolic, then P posses the retarded and advanced Green's functions Δ^R, Δ^A. They satisfy:

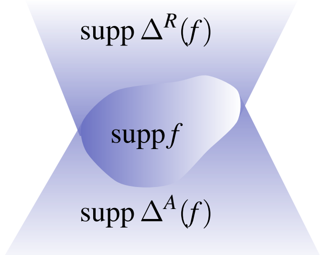
$$P \circ \Delta^{R/A} = \mathrm{id}_{\mathfrak{D}(\mathcal{M})},$$
$$\Delta^{R/A} \circ (P|_{\mathfrak{D}(\mathcal{M})}) = \mathrm{id}_{\mathfrak{D}(\mathcal{M})},$$



Free scalar field

- For the free scalar field the equation of motion is of the form $S'_{\mathcal{M}}(\varphi) = P\varphi = 0$, where $P = -(\Box + m^2)$ is the Klein-Gordon operator.
- If $\mathcal{M}$ is globally hyperbolic, then P posses the retarded and advanced Green's functions Δ^R, Δ^A. They satisfy:

$$P \circ \Delta^{R/A} = \mathrm{id}_{\mathfrak{D}(\mathcal{M})},$$
$$\Delta^{R/A} \circ (P\big|_{\mathfrak{D}(\mathcal{M})}) = \mathrm{id}_{\mathfrak{D}(\mathcal{M})},$$



- Their difference is the causal propagator $\Delta \doteq \Delta^R - \Delta^A$.

Classical theory and deformation

- The classical Poisson bracket is given by

$$\lfloor F, G \rfloor \doteq \left\langle F^{(1)}, \Delta G^{(1)} \right\rangle.$$

Classical theory and deformation

- The classical Poisson bracket is given by

$$\lfloor F, G \rfloor \doteq \left\langle F^{(1)}, \Delta G^{(1)} \right\rangle.$$

- Next: decompose Δ into negative nad positive energy parts (in curved spacetimes use the notion of wavefront sets), i.e.

$$\tfrac{i}{2}\Delta = \Delta_+ - H.$$

Classical theory and deformation

- The classical Poisson bracket is given by
$$\lfloor F, G \rfloor \doteq \left\langle F^{(1)}, \Delta G^{(1)} \right\rangle.$$
- Next: decompose Δ into negative nad positive energy parts (in curved spacetimes use the notion of wavefront sets), i.e.
$$\tfrac{i}{2}\Delta = \Delta_+ - H.$$
- Define the $\star$-product (deformation of the pointwise product) by:

$$(F \star G)(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(n)}(\varphi), (\Delta_+)^{\otimes n} G^{(n)}(\varphi) \right\rangle ,$$

where F, G belong to $\mathfrak{F}_{\mu c}(\mathcal{M})$, a larger class of functionals, which contains the multilocal ones.

Time-ordered product

- Let $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$ be the space of functionals whose derivatives are test functions, i.e. $F^{(n)}(\varphi) \in \mathcal{C}^\infty_c(M^n, \mathbb{R})$,

Time-ordered product

- Let $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$ be the space of functionals whose derivatives are test functions, i.e. $F^{(n)}(\varphi) \in \mathcal{C}^\infty_c(M^n, \mathbb{R})$,
- The time-ordering operator $\mathcal{T}$ is defined as:

$$\mathcal{T}F(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(2n)}(\varphi), (\Delta^F)^{\otimes n} \right\rangle \ ,$$

where $\Delta^F = \frac{i}{2}(\Delta^R + \Delta^A) + H$.

Time-ordered product

- Let $\mathfrak{F}_{\text{reg}}(\mathcal{M})$ be the space of functionals whose derivatives are test functions, i.e. $F^{(n)}(\varphi) \in \mathcal{C}_c^\infty(M^n, \mathbb{R})$,
- The time-ordering operator $\mathcal{T}$ is defined as:

$$\mathcal{T}F(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(2n)}(\varphi), (\Delta^F)^{\otimes n} \right\rangle ,$$

where $\Delta^F = \frac{i}{2}(\Delta^R + \Delta^A) + H$.
- Formally it would correspond to the operator of convolution with the oscillating Gaussian measure "with covariance $i\hbar\Delta^F$",

$$\mathcal{T}F(\varphi) \overset{\text{formal}}{=} \int F(\varphi - \phi)\, d\mu_{i\hbar\Delta^F}(\phi)\,.$$

Time-ordered product

- Let $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$ be the space of functionals whose derivatives are test functions, i.e. $F^{(n)}(\varphi) \in \mathcal{C}_c^\infty(M^n, \mathbb{R})$,
- The time-ordering operator $\mathcal{T}$ is defined as:

$$\mathcal{T}F(\varphi) \doteq \sum_{n=0}^{\infty} \frac{\hbar^n}{n!} \left\langle F^{(2n)}(\varphi), (\Delta^F)^{\otimes n} \right\rangle ,$$

where $\Delta^F = \frac{i}{2}(\Delta^R + \Delta^A) + H$.
- Formally it would correspond to the operator of convolution with the oscillating Gaussian measure "with covariance $i\hbar\Delta^F$",

$$\mathcal{T}F(\varphi) \stackrel{\text{formal}}{=} \int F(\varphi - \phi)\, d\mu_{i\hbar\Delta^F}(\phi) .$$

- We define the time-ordered product $\cdot_{\mathcal{T}}$ on $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})[[\hbar]]$ by:

$$F \cdot_{\mathcal{T}} G \doteq \mathcal{T}(\mathcal{T}^{-1}F \cdot \mathcal{T}^{-1}G)$$

Interaction

- We now have two products on $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})[[\hbar]]$: non-commutative $\star$ and commutative $\cdot_{\mathcal{T}}$. They are related by a relation:

$$F \cdot_{\mathcal{T}} G = F \star G\,,$$

if the support of F is later than the support of G.

Interaction

- We now have two products on $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})[[\hbar]]$: non-commutative $\star$ and commutative $\cdot_{\mathcal{T}}$. They are related by a relation:

$$F \cdot_{\mathcal{T}} G = F \star G\,,$$

if the support of F is later than the support of G.

- Interaction is a functional V (for a moment we assume that it belongs to $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$). Using the commutative product $\cdot_{\mathcal{T}}$ we define the formal S-matrix:

$$\mathcal{S}(V) \doteq e_{\mathcal{T}}^{iV/\hbar} = \mathcal{T}\left(e^{\mathcal{T}^{-1}(iV/\hbar)}\right).$$

Interaction

- We now have two products on $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})[[\hbar]]$: non-commutative $\star$ and commutative $\cdot_{\mathcal{T}}$. They are related by a relation:

$$F \cdot_{\mathcal{T}} G = F \star G\,,$$

if the support of F is later than the support of G.

- Interaction is a functional V (for a moment we assume that it belongs to $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$). Using the commutative product $\cdot_{\mathcal{T}}$ we define the formal S-matrix:

$$\mathcal{S}(V) \doteq e_{\mathcal{T}}^{iV/\hbar} = \mathcal{T}\left(e^{\mathcal{T}^{-1}(iV/\hbar)}\right).$$

- Interacting fields are defined by the formula of Bogoliubov:

$$R_V(F) \doteq \left(e_{\mathcal{T}}^{iV/\hbar}\right)^{\star-1} \star \left(e_{\mathcal{T}}^{iV\hbar} \cdot_{\mathcal{T}} F\right).$$

Interaction

- We now have two products on $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})[[\hbar]]$: non-commutative $\star$ and commutative $\cdot_{\mathcal{T}}$. They are related by a relation:

$$F \cdot_{\mathcal{T}} G = F \star G\,,$$

  if the support of F is later than the support of G.
- Interaction is a functional V (for a moment we assume that it belongs to $\mathfrak{F}_{\mathrm{reg}}(\mathcal{M})$). Using the commutative product $\cdot_{\mathcal{T}}$ we define the formal S-matrix:

$$\mathcal{S}(V) \doteq e_{\mathcal{T}}^{iV/\hbar} = \mathcal{T}\left(e^{\mathcal{T}^{-1}(iV/\hbar)}\right).$$

- Interacting fields are defined by the formula of Bogoliubov:

$$R_V(F) \doteq \left(e_{\mathcal{T}}^{iV/\hbar}\right)^{\star-1} \star \left(e_{\mathcal{T}}^{iV\hbar} \cdot_{\mathcal{T}} F\right).$$

- Renormalization problem: extend these structures to local non-linear functionals (these are not regular…).

Perspectives in pAQFT

Recent results and perspectives:

- Construction of thermal states and introduction of the mollified Hamiltonian formalism ([Fredenhagen-Lindner 13]) opens up a perspective for applications to problems like Bose-Einstein condensation, Lamb shift, etc.

Perspectives in pAQFT

Recent results and perspectives:

- Construction of thermal states and introduction of the mollified Hamiltonian formalism ([Fredenhagen-Lindner 13]) opens up a perspective for applications to problems like Bose-Einstein condensation, Lamb shift, etc.
- Investigation of topological aspects of LCQFT ([Becker-Schenkel-Szabo 14]) is a first step in generalizing the framework towards “LCQFT up to homotopy”.

Perspectives in pAQFT

Recent results and perspectives:

- Construction of thermal states and introduction of the mollified Hamiltonian formalism ([Fredenhagen-Lindner 13]) opens up a perspective for applications to problems like Bose-Einstein condensation, Lamb shift, etc.
- Investigation of topological aspects of LCQFT ([Becker-Schenkel-Szabo 14]) is a first step in generalizing the framework towards "LCQFT up to homotopy".
- Recent results on the Sine Gordon model ([Bahns-KR 16, to appear soon...]) go beyond perturbation theory in showing convergence of the formal S-matrix in the finite regime of the theory.

Perspectives in pAQFT

Recent results and perspectives:

- Construction of thermal states and introduction of the mollified Hamiltonian formalism ([Fredenhagen-Lindner 13]) opens up a perspective for applications to problems like Bose-Einstein condensation, Lamb shift, etc.
- Investigation of topological aspects of LCQFT ([Becker-Schenkel-Szabo 14]) is a first step in generalizing the framework towards "LCQFT up to homotopy".
- Recent results on the Sine Gordon model ([Bahns-KR 16, to appear soon...]) go beyond perturbation theory in showing convergence of the formal S-matrix in the finite regime of the theory.
- Now to quantum gravity...

Ways around some problems in QG

Based on: R. Brunetti, K. Fredenhagen, KR, *Quantum gravity from the point of view of locally covariant quantum field theory*, [arXiv:1306.1058], CMP 2016.

Ways around some problems in QG

Based on: R. Brunetti, K. Fredenhagen, KR, *Quantum gravity from the point of view of locally covariant quantum field theory*, [arXiv:1306.1058], CMP 2016.

- **Non-renormalizability**: use Epstein-Glaser renormalization to obtain finite results for any fixed energy scale. Think of the theory as an effective theory.

Ways around some problems in QG

Based on: R. Brunetti, K. Fredenhagen, KR, *Quantum gravity from the point of view of locally covariant quantum field theory*, [arXiv:1306.1058], CMP 2016.

- **Non-renormalizability**: use Epstein-Glaser renormalization to obtain finite results for any fixed energy scale. Think of the theory as an effective theory.
- **Dynamical nature of spacetime**: make a split of the metric into background and perturbation, quantize the perturbation as a quantum field on a curved background, show background independence at the end.

Ways around some problems in QG

Based on: R. Brunetti, K. Fredenhagen, KR, *Quantum gravity from the point of view of locally covariant quantum field theory*, [arXiv:1306.1058], CMP 2016.

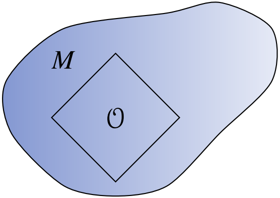
- **Non-renormalizability**: use Epstein-Glaser renormalization to obtain finite results for any fixed energy scale. Think of the theory as an effective theory.
- **Dynamical nature of spacetime**: make a split of the metric into background and perturbation, quantize the perturbation as a quantum field on a curved background, show background independence at the end.
- **Diffeomorphism invariance**: use the BV formalism to perform the quantization.

Intuitive idea

- In experiment, geometric structure is probed by local observations. We have the following data:

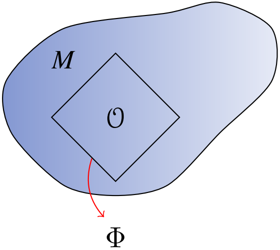
Intuitive idea

- In experiment, geometric structure is probed by local observations. We have the following data:
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,



Intuitive idea

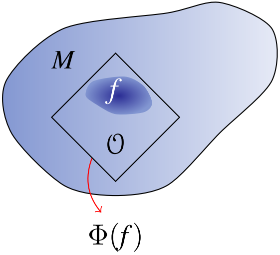
- In experiment, geometric structure is probed by local observations. We have the following data:
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,
  - An observable Φ, which we measure,



Intuitive idea

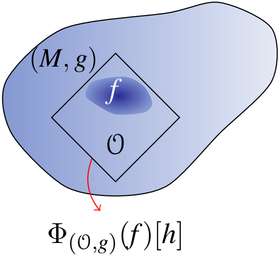
- In experiment, geometric structure is probed by local observations. We have the following data:
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,
  - An observable Φ, which we measure,
  - We don't measure the observable (e.g. curvature) at a point. This is modeled by smearing with a test function f. For example:

$$\Phi(f) = \int f(x) R(x) d\mu(x).$$



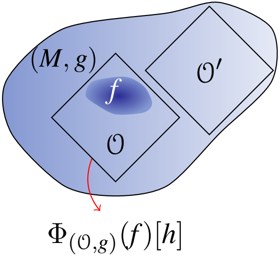
Intuitive idea

- In experiment, geometric structure is probed by local observations. We have the following data:
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,
  - An observable Φ, which we measure,
  - We don't measure the observable (e.g. curvature) at a point. This is modeled by smearing with a test function f. For example:
    $$\Phi(f) = \int f(x)R(x)d\mu(x).$$
- Think of the measured observable as a function of a perturbation of the fixed background metric: $g = g_0 + h$. Hence $\mathfrak{E}(\mathcal{M}) = \Gamma((T^*M)^{\otimes_s 2})$.



Intuitive idea

- In experiment, geometric structure is probed by local observations. We have the following data:
  - Some region $\mathcal{O}$ of spacetime where the measurement is performed,
  - An observable Φ, which we measure,
  - We don't measure the observable (e.g. curvature) at a point. This is modeled by smearing with a test function f. For example:
    $\Phi(f) = \int f(x)R(x)d\mu(x)$.
- Think of the measured observable as a function of a perturbation of the fixed background metric: $g = g_0 + h$. Hence $\mathfrak{E}(\mathcal{M}) = \Gamma((T^*M)^{\otimes_s 2})$.
- Diffeomorphism transformation: move our experimental setup to a different region $\mathcal{O}'$.



Local symmetries in gravity

- In gravity symmetries arise from vector fields on M i.e. we have $\rho : \mathfrak{X}(\mathcal{M}) \to \Gamma(T\mathfrak{E}(\mathcal{M}))$, defined by

$$\partial_{\rho(\xi)} F(h) \doteq \left\langle F^{(1)}(h), \pounds_{\xi}(g_0 + h) \right\rangle ,$$

where $\xi(h) \in \mathfrak{X}(\mathcal{M})$.

Local symmetries in gravity

- In gravity symmetries arise from vector fields on M i.e. we have $\rho : \mathfrak{X}(\mathcal{M}) \to \Gamma(T\mathfrak{E}(\mathcal{M}))$, defined by

$$\partial_{\rho(\xi)} F(h) \doteq \left\langle F^{(1)}(h), \pounds_\xi (g_0 + h) \right\rangle ,$$

where $\xi(h) \in \mathfrak{X}(\mathcal{M})$.

- A locally covariant field A is called diffeomorphism equivariant if it is realized as $A_{(M,g_0)}(f)[h] = \int_M A_g(x) f(x)$, where A_g is a scalar depending locally and covariantly on the full metric, so $A_{\chi^* g} = A_g \circ \chi$ for all formal diffeomorphisms χ.

Local symmetries in gravity

- In gravity symmetries arise from vector fields on M i.e. we have $\rho : \mathfrak{X}(\mathcal{M}) \to \Gamma(T\mathfrak{E}(\mathcal{M}))$, defined by
$$\partial_{\rho(\xi)} F(h) \doteq \left\langle F^{(1)}(h), \pounds_{\xi}(g_0 + h) \right\rangle ,$$
where $\xi(h) \in \mathfrak{X}(\mathcal{M})$.
- A locally covariant field A is called diffeomorphism equivariant if it is realized as $A_{(M,g_0)}(f)[h] = \int_M A_g(x) f(x)$, where A_g is a scalar depending locally and covariantly on the full metric, so $A_{\chi^* g} = A_g \circ \chi$ for all formal diffeomorphisms χ.
- One can characterize the space of classical gauge invariant on-shell observables as the 0th cohomology of the classical BV operator s acting on functionals that satisfy a weaker notion of locality.

Construction of gauge-invariant observables I

- We fix $\mathcal{M} = (M, g_0)$. We want to construct functionals that describe relations between classical fields (relational observables).

Construction of gauge-invariant observables I

- We fix $\mathcal{M} = (M, g_0)$. We want to construct functionals that describe relations between classical fields (relational observables).
- Idea: Take an equivariant field A_g and express it in terms of a coordinate system that depends on other fields.

Construction of gauge-invariant observables I

- We fix $\mathcal{M} = (M, g_0)$. We want to construct functionals that describe relations between classical fields (relational observables).
- Idea: Take an equivariant field A_g and express it in terms of a coordinate system that depends on other fields.
- We realize the choice of a coordinate system by constructing four equivariant fields X^μ_g, $\mu = 0, \dots, 3$ which will parametrize points of spacetime.

Construction of gauge-invariant observables I

- We fix $\mathcal{M} = (M, g_0)$. We want to construct functionals that describe relations between classical fields (relational observables).
- Idea: Take an equivariant field A_g and express it in terms of a coordinate system that depends on other fields.
- We realize the choice of a coordinate system by constructing four equivariant fields X_g^μ, $\mu = 0, \dots, 3$ which will parametrize points of spacetime.
- We choose a background g_0 such that the map

$$X_{g_0} : x \mapsto (X_{g_0}^0, \dots, X_{g_0}^3)$$

is injective.

Construction of gauge-invariant observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

$$\alpha_g = X_g^{-1} \circ X_{g_0} \, .$$

Construction of gauge-invariant observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

$$\alpha_g = X_g^{-1} \circ X_{g_0} \, .$$

- α_g transforms as $\alpha_{\chi^* g} = \chi^{-1} \circ \alpha_g$.

Construction of gauge-invariant observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

$$\alpha_g = X_g^{-1} \circ X_{g_0} \,.$$

- α_g transforms as $\alpha_{\chi^* g} = \chi^{-1} \circ \alpha_g$.
- Let A_g be an equivariant field. Then

$$\mathcal{A}_g := A_g \circ \alpha_g = A_g \circ X_g^{-1} \circ X_{g_0} \,.$$

is invariant under formal diffeo.'s.

Construction of gauge-invariant observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

$$\alpha_g = X_g^{-1} \circ X_{g_0} \, .$$

- α_g transforms as $\alpha_{\chi^* g} = \chi^{-1} \circ \alpha_g$.
- Let A_g be an equivariant field. Then

$$\mathcal{A}_g := A_g \circ \alpha_g = A_g \circ X_g^{-1} \circ X_{g_0} \, .$$

  is invariant under formal diffeo.'s.
- $[A_g \circ X_g^{-1}](Y)$ corresponds to the value of the quantity A_g provided that the quantity X_g has the value $X_g = Y$. Thus it is a partial or relational observable (cf. Rovelli, Dittrich, Thiemann).

Construction of gauge-invariant observables II

- Take $g = g_0 + h$ sufficiently near to g_0 and set

$$\alpha_g = X_g^{-1} \circ X_{g_0}\,.$$

- α_g transforms as $\alpha_{\chi^* g} = \chi^{-1} \circ \alpha_g$.
- Let A_g be an equivariant field. Then

$$\mathcal{A}_g := A_g \circ \alpha_g = A_g \circ X_g^{-1} \circ X_{g_0}\,.$$

  is invariant under formal diffeo.'s.
- $[A_g \circ X_g^{-1}](Y)$ corresponds to the value of the quantity A_g provided that the quantity X_g has the value $X_g = Y$. Thus it is a partial or relational observable (cf. Rovelli, Dittrich, Thiemann).
- By considering $\mathcal{A}_g = A_g \circ X_g^{-1} \circ X_{g_0}$ and choosing a test density f, we identify this observable with a field on spacetime:

$$F = \int \mathcal{A}_g(x) f(x)\,.$$

Examples:

- On generic backgrounds g_0 one can use traces of the powers of the Ricci operator:

$$X_g^a := \mathrm{Tr}(\boldsymbol{R}^a), \qquad a \in \{1,2,3,4\}$$

Examples:

- On generic backgrounds g_0 one can use traces of the powers of the Ricci operator:

$$X_g^a := \mathrm{Tr}(\boldsymbol{R}^a), \qquad a \in \{1,2,3,4\}$$

- More examples: [Bergmann 61, Bergmann-Komar 60].

Examples:

- On generic backgrounds g_0 one can use traces of the powers of the Ricci operator:

$$X_g^a := \mathrm{Tr}(\boldsymbol{R}^a), \qquad a \in \{1,2,3,4\}$$

- More examples: [Bergmann 61, Bergmann-Komar 60].
- When matter fields are present in the considered model, also these can serve as coordinates, e.g. the dust fields in the Brown-Kuchař model [Brown-Kuchař 95]; the scalar field in the Einstein-Klein-Gordon system.

Examples:

- On generic backgrounds g_0 one can use traces of the powers of the Ricci operator:

$$X_g^a := \mathrm{Tr}(\boldsymbol{R}^a), \qquad a \in \{1,2,3,4\}$$

- More examples: [Bergmann 61, Bergmann-Komar 60].
- When matter fields are present in the considered model, also these can serve as coordinates, e.g. the dust fields in the Brown-Kuchař model [Brown-Kuchař 95]; the scalar field in the Einstein-Klein-Gordon system.
- For an explicit construction on a cosmological background see the recent work by R. Brunetti, K. Fredenhagen, T.-P. Hack, N. Pinnamonti and myself: *Cosmological perturbation theory and quantum gravity* [arXiv:gr-qc/1605.02573], JHEP 2016.

Conclusions

- More that 60 years after the paper of Haag and Kastler the locality principle is a powerful paradigm in QFT.

Conclusions

- More that 60 years after the paper of Haag and Kastler the locality principle is a powerful paradigm in QFT.
- It turned out to be very successful in QFT on curved spacetime and it pointed a direction towards effective theory of quantum gravity.

Conclusions

- More that 60 years after the paper of Haag and Kastler the locality principle is a powerful paradigm in QFT.
- It turned out to be very successful in QFT on curved spacetime and it pointed a direction towards effective theory of quantum gravity.
- Combined with perturbative methods it gave rise to pAQFT.

Conclusions

- More that 60 years after the paper of Haag and Kastler the locality principle is a powerful paradigm in QFT.
- It turned out to be very successful in QFT on curved spacetime and it pointed a direction towards effective theory of quantum gravity.
- Combined with perturbative methods it gave rise to pAQFT.
- It is also interesting and challenging to look at possible generalizations: non-local quantities in QG, LCQFT up to homotopy.

Conclusions

- More that 60 years after the paper of Haag and Kastler the locality principle is a powerful paradigm in QFT.
- It turned out to be very successful in QFT on curved spacetime and it pointed a direction towards effective theory of quantum gravity.
- Combined with perturbative methods it gave rise to pAQFT.
- It is also interesting and challenging to look at possible generalizations: non-local quantities in QG, LCQFT up to homotopy.
- More to come!

Thank you for your attention!